CONCOURS D'ADMISSION 2018 – FILIERE UNIVERSITAIRE INTERNATIONALE

FALL SESSION

PHYSICS

(Duration : 2 hours)

The three problems are independent. If you are not able to solve a question, we advise you to assume the result of that question and to move to the next one. The different questions are, to a large extent, independent of each other.

The use of electronic calculators is forbidden.

I. Propagation in a coaxial cable

The aim of this problem is to find relations between the physical parameters of a coaxial cable that optimize the propagation of potential waves.

A coaxial cable (Figure 1) is made of :

(i) A copper wire, with a radius a and conductivity γ ;

(ii) An insulation material with a dielectric constant ε_r and leak conductivity γ_r ;

(iii) A copper mesh with the same conductivity γ as the wire, with a radius b and negligible thickness ;

(iv) An outside insulation.

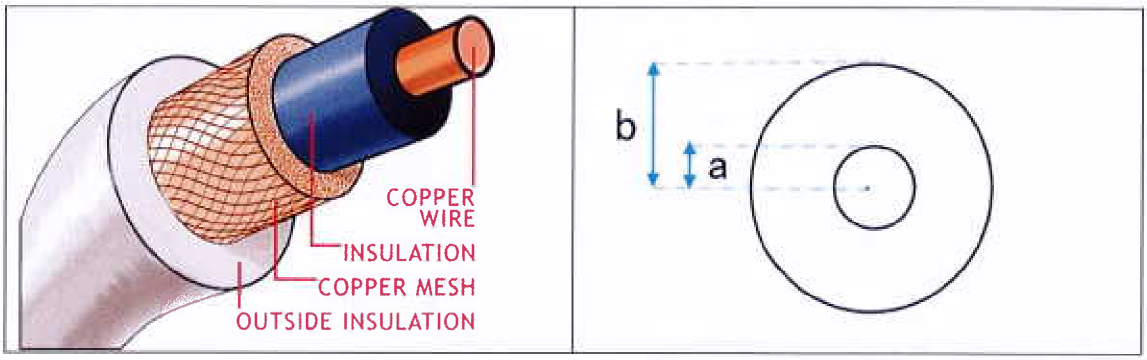

Figure 1. Left : constitution of a coaxial cable. Right : sectional view of the cable (see text for details).

1) Modeling of a coaxial cable.

Compute the following electrical quantities as a function of a, b, γ , γ_r , ε_r and the fundamental electric constants ε_0 and μ_0 :

a) the resistance per unit length of the copper wire, denoted R ;

b) the leak conductance per unit length of cable, denoted G ; note : for this computation you may assume that the copper wire is at potential V and the copper mesh at zero potential, then compute the electrical field between the copper wire and the copper mesh, hence deduce the leaking electric current intensity flowing through the insulation material ;

c) the capacity per unit length of cable, denoted C ;

d) the self-inductance per unit length of cable, denoted L ; note : for this computation, it may be useful to remember that the volume density of magnetic energy is $\frac{B^2}{2\mu_0}$.

2) Wave equation for the electric potential (« telegraph equations »).

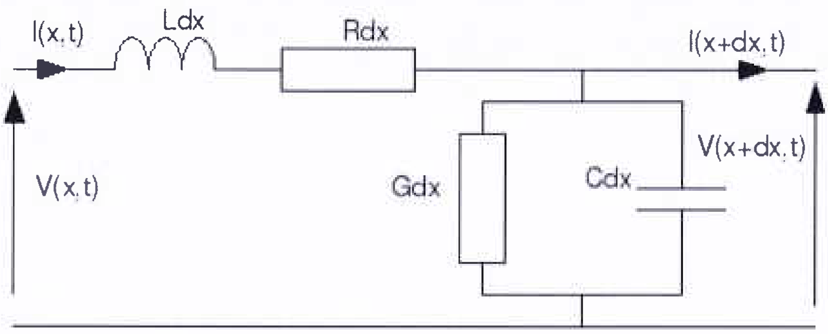

Figure 2. Electric diagram equivalent to an infinitesimal segment of coaxial cable of length dx. V(x,t) is the tension between the internal wire and the copper mesh (which is connected to the ground, hence tied to zero potential). V(x,t) is therefore the electric potential of the internal wire at position x and time t. I(x,t) is the current intensity in the internal copper wire.
Beware : whereas Rdx is the *resistance* of an infinitesimal segment of copper wire, Gdx is the *conductance* of an infinitesimal segment of insulation material.

Find two partial derivative equations that are satisfied by V(x,t) and I(x,t).

3) Hence deduce an equation for V(x,t).

4) We look for solutions in the form $V(x,t) = V_0 e^{j(\omega t - kx)}$ where j is, as usual, the unit imaginary number. Derive the following relationship between ω and k :

$$k^2 = LC\omega^2 - j(RC + GL)\omega - RG$$

5) Quasi stationary regime.

Let's assume that $GL \ll RC$ and $R \gg L\omega$.

a) Prove that the partial differential equation for V(x,t) now writes

$$\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial V}{\partial t} + RGV$$

b) Show that $V(x,t) = V_0 e^{j(\omega t - k'x)} e^{k''x}$ is a solution of this equation with k' and k'' two real constants to be determined. Assuming that $G \gg C\omega$, give k' as a function of R, G, C and ω, and k'' as a function of R and G.

c) What is the propagation speed (phase velocity) of the potential wave $V(x,t)$?

6) High frequency regime.

Let's assume now that $R \ll L\omega$ and $G \ll C\omega$. We note $LC = \frac{1}{c_0^2}$.

a) Compute the real and imaginary parts of the wave vector k.

b) What is the meaning of c_0?

c) Show that the amplitude of the potential wave decreases exponentially with the distance x along the cable. Express the characteristic distance of damping λ as a function of L, C, R, G.

7) Heaviside condition.

Show that the cable is non dispersive if and only if the following condition, known as « Heaviside condition », is satisfied:

$$\frac{L}{C} = \frac{R}{G}$$

We recall that a nondispersive medium is a medium where the phase velocity of the propagating waves is independent of the wave frequency. What is the advantage of a nondispersive medium ?

II. Laying cables under the sea : mechanical problems

Most intercontinental communications are still going through undersea cables. Specialized ships are used to lay cables under the sea : such ships have a cable winch (big spool) at their stern (rear end of the ship) around which the cable is coiled. As the ship is moving, the cable is unwound from the winch. A diagram of the cable shape underwater is given in Figure 3.

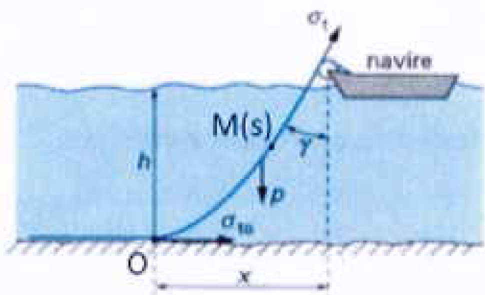

Figure 3. Diagram of a ship (« navire » in french) laying a cable under the sea. Note the cable winch at the stern (rear end of the ship). At the end of the cable winch, the tangent line to the cable is at an angle γ with the vertical line. The *apparent* linear weigth density (weight of the unit length of cable minus Archimedes' buoyant force) is denoted p.

Let s denote the curvilinear abscissa along the cable, i.e. the length of cable between the origin O and the point M(s). We note $\alpha(s)$ the angle between the tangent to the cable at point M(s) and the vertical line.

1) Let λ be the linear mass density of the cable and D the cable diameter. Usual cables have $\lambda = 10\ kg.m^{-1}$ and $D = 70\ mm$. Evaluate the *apparent* linear weigth density p (see Figure 3 and its legend).

2) The ship is assumed to be at rest. Write the mechanical equilibrium of an infinitesimal segment of cable of length ds.

3) Prove that the tension σ_t everywhere along the cable satisfies the two following equations :

$$\sigma_t sin\alpha = \sigma_{t0}$$
$$\sigma_t cos\alpha = ps$$

4) Hence deduce the following relation between x and h (see Figure 3) :

$$h = \frac{\sigma_{t0}}{p}\left[ch\left(\frac{px}{\sigma_{t0}}\right) - 1\right]$$

5) The ship now moves at a constant speed V. Assuming that V is low enough, explain why the equilibrium equations derived in questions 2 and 3 are still satisfied. What is the physical meaning of « low enough » ?

6) At this speed V, the ship engine exerts a force F on the ship. Compute the angle γ between the cable and the vertical line at the end of the cable winch (see Figure 3) as a function of F, p and h.

III. A clever measuring cup

In order to improve measuring for cooking, engineers have designed a new shape of measuring cup (see Figure 4).

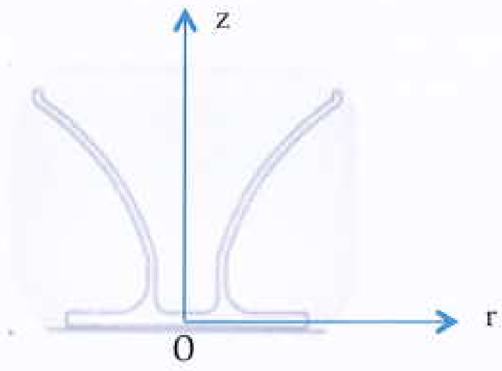

Figure 4 : vertical sectional view of the new shape of measuring cup. Note that Oz is a rotational axis of symmetry of the measuring cup.

The engineers argue that the best shape must fulfill the following equation:

$$\frac{A(z)}{\int_0^z A(u)du} = k \qquad \text{(III.1)}$$

where k is a constant. A(z) is the area of the horizontal sectional view at height z.

1) What is the rationale for this equation ?
2) What is the meaning of the constant k ? Give an estimate for k.
3) Find out a solution of Equation III.1 giving r as a function of z.