



CONCOURS D'ADMISSION 2017
AUTUMN SESSION

FILIÈRE UNIVERSITAIRE INTERNATIONALE

PHYSICS

(Duration : 2 hours)

*The problem and the exercise are independent and can be solved in any order.
Make sure to carry out numerical calculations when requested and round numerical
values to one significant digit. The use of electronic calculators is forbidden.*

Problem – Water purification

The purpose of this problem is to study the removal of small solid particles from liquid water as part of the water purification process. We study two methods used for this purpose : decantation (questions 1 to 4) and coagulation (questions 5 to 16).

Decantation consists in waiting that the particles fall to the bottom of the container through the action of gravity which acceleration is denoted by \vec{g} .

1. Use Archimedes' principle to evaluate the buoyant (upward) force \vec{F}_A exerted on a solid particle of volume V and density (volumetric mass density) μ_s surrounded by a static liquid of density μ . Under what condition does the particle fall to the bottom ?

A spherical particle of radius R falling with velocity \vec{v} in a fluid of kinematic viscosity ν experiences a drag force given by Stokes' law : $\vec{F}_d = -6\pi\nu\mu R\vec{v}$.

2. What is the dimension of ν ?

3. Show that the falling particle reaches a maximum velocity v_d , the so-called decantation velocity. Express v_d as a function of R , ν , g and of the ratio μ_s/μ .

4. What condition must v_d satisfy in order for decantation to be efficient after a duration $\Delta t = 1$ day in a container of height $h = 1$ m ? What is the resulting condition on the radius R of particles whose density is twice that of water, $\mu_s = 2\mu$? Express the result first algebraically, then numerically ; kinematic viscosity of water is $\nu = 10^{-6}$ SI.

We now study an alternative process named coagulation, which is the aggregation of small particles into larger particles, which are eventually eliminated by decantation. Aggregation is triggered by the van der Waals attraction between two neighboring particles.

But this attraction is counterbalanced by the electrostatic repulsion between particles, which carry an electric charge on their surface. We assume for simplicity that the charge density per unit surface σ is positive and identical for all particles. In order to induce coagulation, one must first decrease this electrostatic repulsion. This is achieved by adding ions of opposite charges q et $-q$ to the water, with $q > 0$. The densities of positive and negative ions at infinity are identical and denoted by n (number density per unit volume).

5. Describe qualitatively how the concentrations of positive and negative ions are modified in the vicinity of the surface of a particle.

We now carry out a more quantitative study of the electric potential near the surface of a particle. We model this surface as an infinite plane Oyz . The particle occupies the half-space $x < 0$, the water occupies the half-space $x > 0$.

6. Write the differential equation relating the electric potential $V(x)$ to the charge density $\rho(x)$ inside the liquid.

Hint : The electric field \vec{E} is related to the charge density ρ by $\text{div} \vec{E} = \rho/\epsilon$, where ϵ denotes the dielectric permittivity of the water.

7. What is the potential energy $U_+(x)$ of a positive ion at point x ? What is the potential energy $U_-(x)$ of a negative ion at point x ?

8. Explain why the number density of positive or negative ions at point x is given by $n_{\pm}(x) = n \exp(-U_{\pm}(x)/k_B T)$, where T is the temperature of the solution.

9. Use this result to express the *total* charge density $\rho(x)$ as a function of $V(x)$.

10. Write the differential equation satisfied by $V(x)$ for $x > 0$. Linearize this equation when $|U_{\pm}(x)| \ll k_B T$. Show that a characteristic length λ appears in this equation.

11. Solve the differential equation for $V(x)$ and sketch its graph.

12. Use Gauss's flux theorem to express the electric field at $x = 0^+$ as a function of charge density per unit surface σ . Use this result to relate $V(x = 0^+)$ and σ . What is the electrical analog of this situation?

13. Explain how the differential equation satisfied by $V(x)$ can be solved if the condition $|U_{\pm}(x)| \ll k_B T$ is released. Describe your method without doing all the calculation.

Hint : Use a mechanical analogy with the equation of motion of a particle under a conservative force.

14. Using previous results, justify briefly why the electrostatic energy between two solid particles whose mutual distance x is much smaller than their size can be written as

$$U_E(x) = B \exp\left(-\frac{x}{\lambda}\right)$$

where B is a positive constant.

In addition, the particles interact through the Van der Waals force which associated potential energy is given by $U_V(x) = -A/x^2$ where A is a positive constant. The total interaction energy is thus $U(x) = U_E(x) + U_V(x)$.

15. Determine under which condition $U(x) \leq 0$ for all $x > 0$.

Hint : Introduce a dimensionless parameter β proportional to B and discuss the relative importance of each energy as a function of β .

16. Under which condition on β does coagulation occur?

Exercise – Nucleation

When the temperature of a liquid is lowered below its freezing point, it turns into a solid through the nucleation of small grains. While the transformation of the liquid into a solid is energetically favored below the freezing point, the formation of a grain comes with an additional cost due to the surface tension between the solid and the liquid. The purpose of this exercise is to study the competition between these two effects.

1. Recall why the Gibbs energy (or free enthalpy) G is the relevant thermodynamic potential at fixed pressure and temperature.
2. We denote by μ_S and μ_L the Gibbs energy per mole in the solid and liquid phase, respectively. What is the sign of $\mu_S - \mu_L$ below the freezing point?

The density (volumetric mass density) of the solid phase is ρ_S and the molar mass is M . We recall that the surface tension γ of the interface between the solid and the liquid is defined as the work $dW = \gamma dA$ needed to increase its surface area by dA at fixed temperature and pressure.

3. Write the variation $\Delta G(R)$ associated with the nucleation of a spherical solid grain of radius R . Sketch its variation as a function of R .
4. Discuss the fate of a grain depending on its size.

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